Thus, the theoretical two-dimensional time-dependent solution, Eq. (9), has been tested as a description of the viscous cross flow for large fineness ratio bodies approximating circular cylinders at moderate angles of attack. Some agreement has been obtained, although empirical procedures are still required and the region of applicability is narrow. For slender bodies of high angles of attack, where vortices depart an appreciable distance from the body surface, Eq. (9), a boundary-layer-type solution, would be invalid.

References

[1] Woodward, F. A., "Analysis and Design of Wing-Body Combinations at Subsonic and Supersonic Speeds," *Journal of Aircraft*, Vol. 5, No. 6, Nov.-Dec. 1968, pp. 528–534.

[2] Allen, H. J. and Perkins, E. W., "A Study of Effects of Viscosity on Flow Over Slender Inclined Bodies of Revolution," Rept. 1048, 1951, NACA.

[3] Washington, W. D., "Correlation of Viscous Effects and Comparison Betweeen Experimental and Theoretical Distribution of Potential Normal Force and Pitching Moments for Bodies of Revolution $M > 1$," Rept. RD-TR-12-67, Dec. 1967, Redstone Arsonal, Huntsville, Ala.

[4] Jorgensen, L. H. and Perkins, E. W., "Investigation of Some Wake and Vortex Characteristics of an Inclined Ogive-Cylinder Body at Mach No. 2," Rept. 1371, 1958, NACA; also TN 3716, 1956, NACA.

[5] Wang, C.-Y., "The Flow Past A Circular Cylinder Which is Started Impulsively from Rest," *Journal of Mathematics and Physics*, Vol. 46, 1967, pp. 195–202.

[6] Wundt, H., "Wachstum der laminaren Grenzschict an schräg angeströmeten Zylindern bei Anfahrt aus der Ruhe," *Ingenieur-Archiv*, Vol. 23, 1955, pp. 212–230.

[7] Jones, R., "The Distribution of Normal Pressure on a Prolate Spheroid," R&M 1061, 1925, Aeronautical Research Council, London.

Parametric Shimmy of a Nosegear

F. H. Ho* and J. L. Lai†

B. F. Goodrich Research Center, Brecksville, Ohio

Nomenclature

a = a parameter in a standard Mathieu Equation (15)
a_0, a_2 = coefficients of the characteristic Eq. (4) when $\epsilon = 0$
C = the coefficient of yaw of a tire (rad/lb)
f = force generated by the wheel and tire imperfections, in the wheel plane (lb)
F_n = lateral ground reaction (lb)
F_r = vertical ground reaction (lb)
g = a parameter defined in Eq. (16)
H = distance from trunion to axle (in.)
I_f = moment of inertia of the system about the strut axis (lb-in.-sec²)
I_s = moment of inertia of the swivelling parts about the spindle (lb-in.-sec²)
K_0 = torsional spring constant about the strut axis (in.-lb/rad)
K_t = torsional spring constant about the spindle (in.-lb/rad)
L = trail length (in.)
m = wheel mass (lb-sec²/in.)
M_f = the moment of the force f about the strut axis (in.-lb)
q = a parameter in a standard Mathieu Equation (15)
R = wheel radius (in.)
t = time variable (sec)
u = the amount of unbalances (in.-oz)
V = vehicle velocity (in./sec or mph)
W = weight on the nosegear (lb)
α = the camber angle of the nosegear (rad)
$\epsilon, \epsilon_1, \epsilon_2$ = the product of the force f and the trail length, refer to Eqs. (5) and (12)
θ = the yaw angle of the nosegear (rad)
τ = nondimensional time variable defined in Eq. (13)
ω = the frequency of rotation of the wheel (rad/sec)
ω_1, ω_2 = the fundamental frequencies of the nosegear (rad/sec)

Symbols

[ ] = a square matrix
{ } = a column matrix
$\dot{x}$ = dx/dt

In the previous studies[1–3] of the nosegear shimmy, the effect of wheel and tire imperfections were not discussed. The purpose of this Note is to show that instability of certain nosegear can result from large wheel and tire imperfections. And the phenomenon of this self-excited instability (shimmy) is similar to that of the well-known parametric resonance.[4,5] It is different from the ordinary resonance caused by an external periodic force in that the major instability occurs when the wheel rotation frequency is equal to double any natural frequency of the system. For a simplified nosegear system studied in this Note, we find this parametric shimmy of landing gear is determined completely by the instability character of an associated Mathieu equation.

To avoid great mathematical difficulties, we use a much simplified nosegear model shown in Fig. 1. This model resembles essentially that used by Moreland,[2] and has two degrees of freedom (camber α and yaw θ). In this study, the forces generated by all wheel and tire imperfections, e.g., unbalances, flat spots, etc. are lumped together and represented by a single force f which is the centrifugal force generated by a single unbalance weight located at the periphery of the tire Fig. (1b) thus,

$$f(\omega) \approx (u/6176)\omega^2, \ \omega \approx V/R \tag{1}$$

The quantities in the previous and subsequent equations are defined in the nomenclature. Note in the yawed position

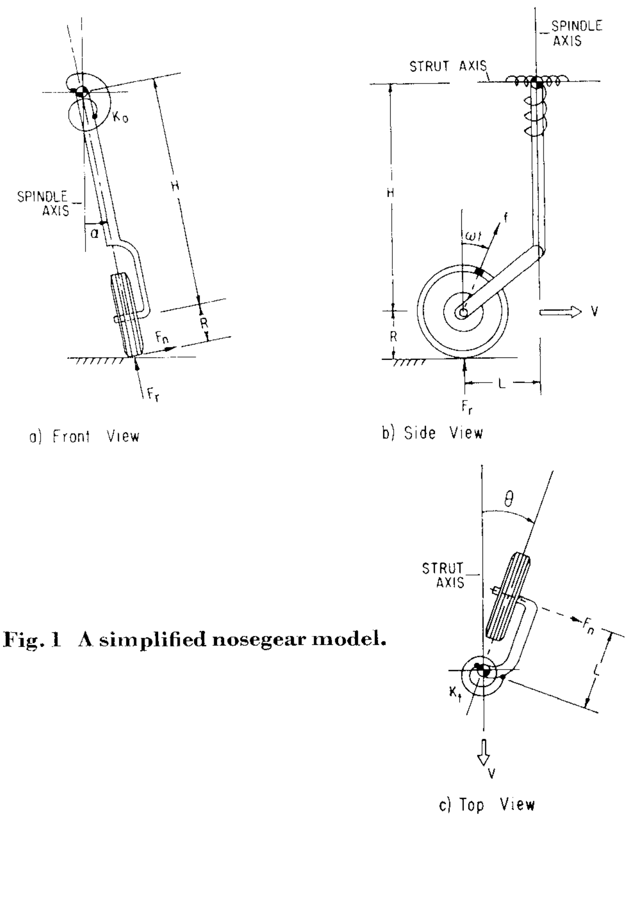

Fig. 1 A simplified nosegear model.

Received February 2, 1970.

* Research Associate, Aerospace Products Research. Member AIAA.

† Senior Engineering Scientist. Member AIAA.

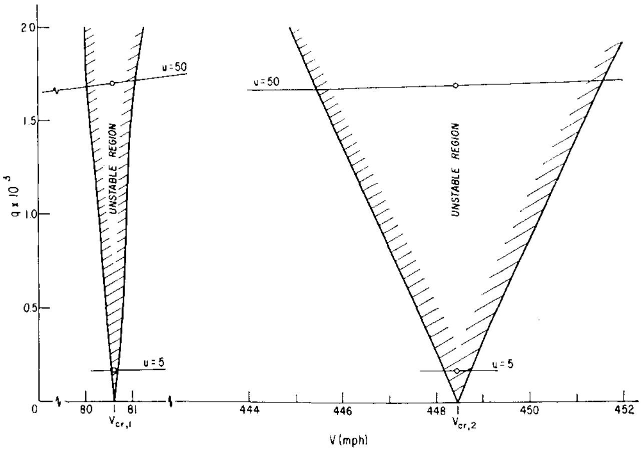

Fig. 2 The regions of instability near the two principal critical velocities.

Fig. 1b and 1c), the force f does not pass through the strut axis. And it has a normal component in the same direction as the ground reaction F_r. This component of f has a moment with respect to the strut axis, for small yaw angles, thus,

$$M_f = (f \cos\omega t)L\theta \tag{2}$$

where L is the trail length. We assume f is in the wheel plane, therefore it has no moment with respect to the spindle axis. Also, to simplify the tire model, we assume the relation between the lateral ground force F_n and the wheel yaw angle θ is

$$\theta = -CF_n \tag{3}$$

Referring to the system in Fig. 1 and neglecting the gyroscopic and pendulous moments,[2] summations of the moments of the forces about the strut and the spindle axes will lead to the following homogeneous governing equations:

$$[I]\{\ddot{x}\} + [K]\{x\} - \epsilon[B]\cos\omega t\{x\} = 0 \tag{4}$$

where

$$\{x\} = \begin{Bmatrix} \alpha \\ \theta \end{Bmatrix} \tag{5a}$$

$$[I] = \begin{bmatrix} I_f & mLH \\ mLH & I_s \end{bmatrix} \tag{5b}$$

$$[K] = \begin{bmatrix} K_0 & (H+R)/C - WL \\ 0 & K_t + L/C \end{bmatrix} \tag{5c}$$

$$[B] = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \tag{5d}$$

$$\epsilon = \epsilon(\omega) = f(\omega)L \tag{5e}$$

Quantities and symbols in Eqs. (4) and (5) are defined in Fig. 1 and in the nomenclature. Note Eq. (4) can be reduced to

$$\{\ddot{x}\} + ([A] - \epsilon[\beta]\cos\omega t)\{x\} = 0 \tag{6}$$

in which

$$[A] = [I]^{-1}[K],\ [\beta] = [I]^{-1}[B] \tag{7}$$

When $\epsilon = 0$, i.e. for a perfect wheel and tire, Eq. (6) can be readily solved, and the stability criterion of the system is

$$a_0 > 0,\ a_2 > 0, \text{ and } (a_2^2 - 4a_0a_4) > 0 \tag{8}$$

where

$$a_0 = I_fI_s - (mLH)^2$$
$$a_2 = I_f(K + L/C) - mLH[(H+R)/C - WL] + K_0I_s \tag{9}$$
$$a_4 = K_0(K_t + L/C)$$

When $\epsilon \neq 0$, the system in Eq. (6) is instationary and we shall discuss its stability by using an approximate technique described by Bolotin.[5] We now try to find the equivalent eigen-system of Eq. (6) thus,

$$\ddot{x}_k + \Omega_k^2(t)x_k = 0,\ (k = 1,2) \tag{10}$$

As a good approximation, we find Ω_k as follows:

$$\Omega_k^2 \approx \omega_k^2 - \epsilon_k g \cos\omega t,\ (k = 1,2) \tag{11}$$

where

$$g = \tfrac{1}{2}\beta_{22},\ \epsilon_k = \epsilon(\omega_k),\ (k = 1,2) \tag{12}$$

In the previous equations ω_1^2 and ω_2^2 are the squares of the fundamental natural frequencies of a perfect nosegear system and β_{22} in Eq. (12) is an element in the $[\beta]$ matrix in Eq. (7).

Using a new time variable τ defined by

$$2\tau = \omega t \tag{13}$$

and substituting Eqs. (11–13) into (10), we obtain Equations

$$\frac{d^2\alpha}{d\tau^2} + \left(\frac{2\omega_1}{\omega}\right)^2\left(1 - \frac{\epsilon_1 g}{\omega_1^2}\cos 2\tau\right)\alpha = 0 \tag{14a}$$

$$\frac{d^2\theta}{d\tau^2} + \left(\frac{2\omega_2}{\omega}\right)^2\left(1 - \frac{\epsilon_2 g}{\omega_2^2}\cos 2\tau\right)\theta = 0 \tag{14b}$$

(14a) and (14b) are standard Mathieu equations[4] of the form

$$\ddot{y} + (a - 16q\cos 2\tau)y = 0 \tag{15}$$

where

$$a = (2\omega_k/\omega)^2,\ q = \epsilon_k g/4\omega^2,\ (k = 1,2) \tag{16}$$

The stability of the Mathieu Equation (15) has been thoroughly investigated.[4,5] We will, therefore, discuss the phenomenon of the parametric shimmy by using the well-known Strutt diagram. For the nosegear model shown in Fig. 1, there are two principal unstable regions near where the values of a in Eq. (16) equal to unity. Therefore, the shimmy of nosegear will occur when the wheel rotation frequency (ω) is twice of either natural frequency (ω_k) of the system. Furthermore, these unstable regions are widened as the wheel-tire imperfections become larger, or as the trail length increases, because these effects will increase the value of q in Eq. (16).

Note there are also other unstable regions in the Strutt diagram near where the a values are greater than unity. Therefore, parametric shimmy is also possible at other lower wheel rotation frequencies. However, since dampers are always used in real nosegear, those other unstable regions are practically less important than the principal instability zone just discussed.

Numerical Example

Let us assume the following values for the nosegear shown in Fig. 1:

$$H = 60 \text{ in.},\ R = 12.5 \text{ in.},\ L = 20 \text{ in.}$$
$$I_f = 2000 \text{ in.-lb-sec}^2,\ I_s = 10 \text{ in.-lb-sec}^2,\ C = 0.0001 \text{ rad/lb} \tag{17}$$
$$m = 0.1 \text{ lb-sec}^2\text{/in.},\ W = 1500 \text{ lb},$$
$$K_0 = 6 \times 10^6 \text{ in.-lb/rad},\ K_t = 10 \times 10^4 \text{ in.-lb/rad}$$

For a perfect wheel and tire system ($\epsilon = 0$), substituting Eq. (17) into Eqs. (8) and (9), the nosegear system is stable and no shimmy will occur. When the wheel-tire imperfections are considered ($\epsilon \neq 0$), the instability of the system is determined by the values of a and q of the Mathieu Eq. (15). Note the natural frequencies of the camber and yaw modes for this system are, respectively, 9 cps and 50 cps.

According to previous discussions, parametric shimmy will occur as the value of a in Eqs. (16) approaches unity, i.e., when the velocity of the vehicle reaches the following two critical values:

$$V_{cr,1} \approx 80.6 \text{ mph}, \; V_{cr,2} \approx 448.5 \text{ mph} \tag{18}$$

The two principal unstable regions of Eq. (14) are shown in Fig. 2. q in Fig. 2 is defined in Eq. (16).

To demonstrate the phenomenon of the parametric shimmy and to evaluate the approximations in Eq. (11), we have numerically integrated the original Eq. (6), by using the values in Eq. (17). The results are given in Figs. 3a and 3b. Figure 3a shows the camber α and shimmy θ motions of the gear when the vehicle travels at its first critical speed $\approx$80.6 mph. Note, at this speed, the frequency of the wheel unbalance force is approximately 18 cps, while the unstable motions of both camber and yaw are oscillating at 9 cps—the fundamental natural frequency of the system. Observed also in Fig. 3a, the shimmy motion θ, at the beginning, has a tendency to oscillate at its own natural frequency (50 cps); but gradually moves with the camber motion as α grows. Similar phenomena are found at the second critical speed ($\approx$448 mph), where α moves with the yaw θ, and the gear is also unstable (Fig. 3b).

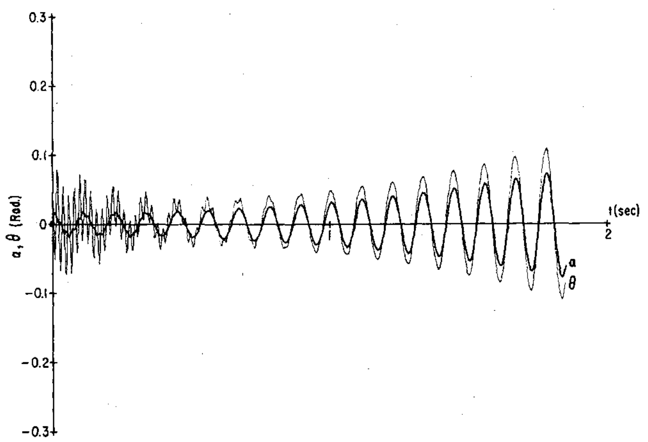

Fig. 3a Shimmy and camber motions of the nosegear (at V = 80.6 mph).

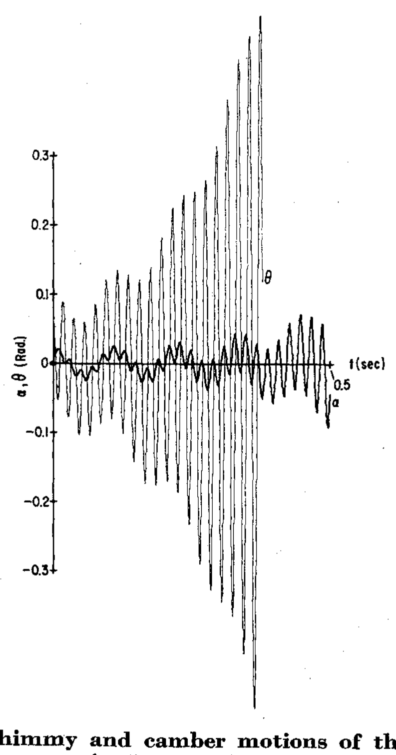

Fig. 3b Shimmy and camber motions of the nosegear (at V = 448.5 mph).

The parametric shimmy shown in Fig. 3a and 3b is obviously of a self-excited type. In a real gear system where damping is used, this shimmy is possible only when the wheel imperfection is so significantly large that the actual q value in Eq. (16) will fall into the instability region of a damped Mathieu equation,[4,5] in the vicinity of the critical speeds. Also in the present gear model, the parametric shimmy will not occur when the trail length L is zero.

References

[1] Von Schlippe, B. and Dietrich, R., "Shimmying of a Pneumatic Wheel," TM-1365, Aug. 1954, NACA, pp. 125–160.

[2] Moreland, W. J., "The Story of Shimmy," *Journal of the Aeronautical Sciences*, Vol. 21, No. 12, Dec. 1954, pp. 793–808.

[3] Smiley, R. F., "Correlation, Evaluation, and Extension of Linearized Theories for Tire Motion and Wheel Shimmy," TN-3632, June 1956, NACA.

[4] Hayashi, C., *Nonlinear Oscillations in Physical Systems*, McGraw-Hill, New York, 1964.

[5] Bolotin, V. V., *The Dynamic Stability of Elastic Systems*, Holden-Day, San Francisco, Calif., 1964.

Lower Bounds for Sonic Boom Considering the Negative Overpressure Region

JAMES S. PETTY*
Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio

THE possible inapplicability of the asymptotic sonic boom theory for large SST-type aircraft, particularly during the critical transonic acceleration flight condition, was noted by McLean.[1] Subsequently, the nonasymptotic theory has been utilized to obtain configurations with considerably lower boom overpressures than earlier believed possible. Reasoning similar to that used by Jones[2,3] may be used to obtain lower bound configurations from the nonasymptotic theory if only the positive overpressure is of interest.[4] However, unlike the asymptotic overpressure signature the nonasymptotic signature is not, in general, antisymmetric about its midpoint and, in particular, the magnitude of the negative peak overpressure may be larger than the positive peak overpressure. Therefore, it is necessary to consider both the positive and negative parts of the overpressure signature when seeking lower bound configurations from the nonasymptotic theory. In the analysis that follows, we draw heavily on the definitive paper of Whitham[5] and on those of Jones and Walkden.[6]

We assume that the aircraft may be replaced by the equivalent area distribution $A_e(x) = S(x) + \beta L(x)/\rho V^2$, for $x < l$, where $S(x)$ is the cross-sectional area distribution, $L(x)$ is the integral of the longitudinal lift distribution, and l is the over-all aircraft length. For $x > l$ the distribution $A_e(x) = A_e(l) \equiv A_{e,b}$ is assumed. If the equivalent distribution is smooth and slender, then A_e and Whitham's function F are related by the Abel transform pair

$$A_e(x) = 4 \int_0^x F(y)(x - y)^{1/2} dx \tag{1}$$

Received January 7, 1970; revision received May 30, 1970.

* Aerospace Engineer, Hypersonic Research Laboratory. Member AIAA.